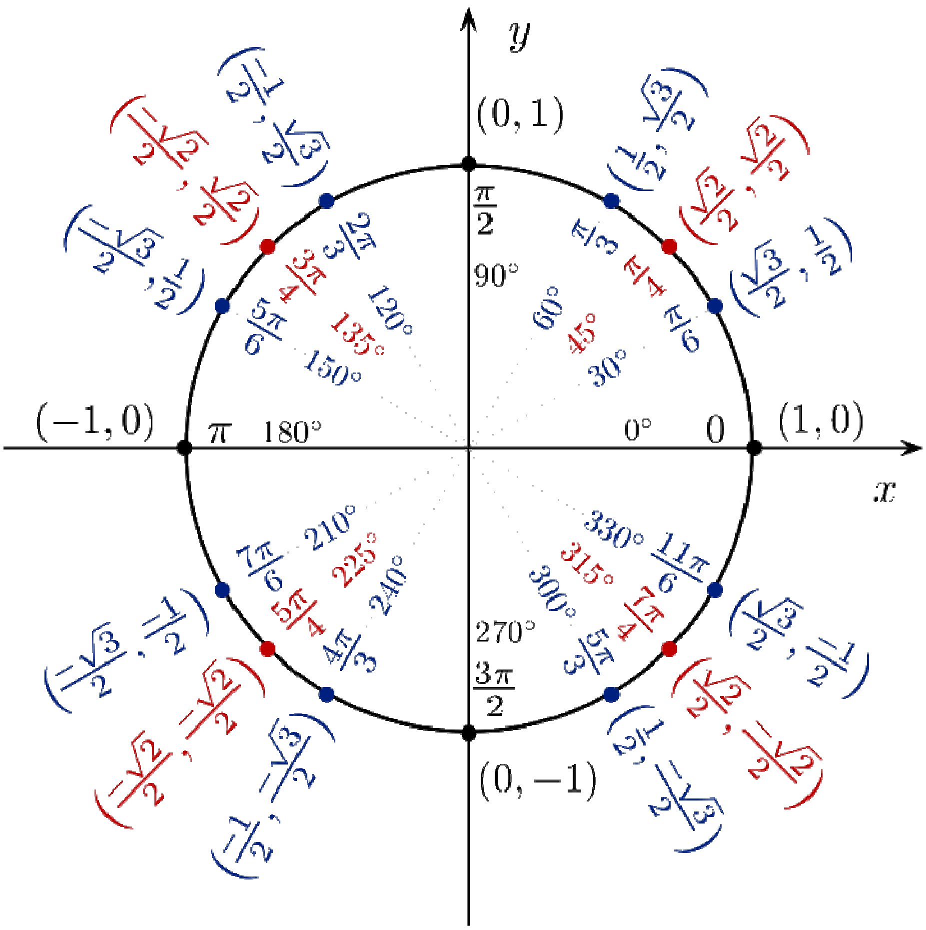
Trigonometry Summary

Right Triangle Trigonometry

$\sin\theta = \dfrac{opposite \quad (y)}{hypotenuse \quad (r)}$ $\qquad \cos\theta = \dfrac{adjacent \quad (x)}{hypotenuse \quad (r)}$ $\qquad \tan\theta = \dfrac{opposite \quad (y)}{adjacent \quad (x)}$

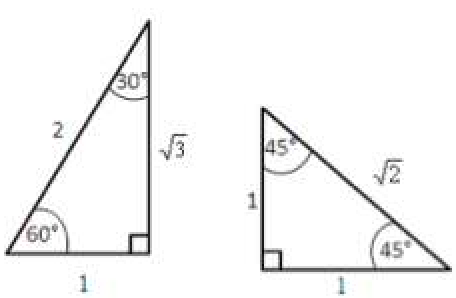
$\csc\theta = \dfrac{hypotenuse \quad (r)}{opposite \quad (y)}$ $\qquad \sec\theta = \dfrac{hypotenuse \quad (r)}{adjacent \quad (x)}$ $\qquad \cot\theta = \dfrac{adjacent \quad (x)}{opposite \quad (y)}$

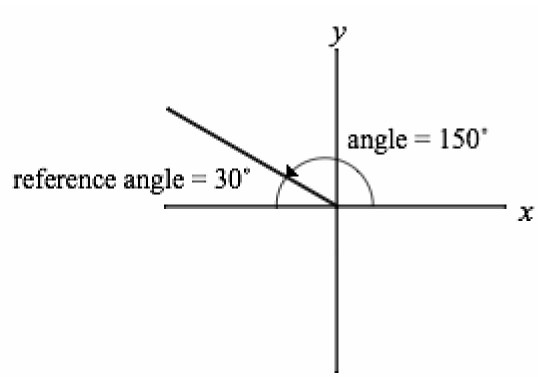
Unit Circle: Radius 1, Coordinates (x, y) on the unit circle are $(\cos\theta, \sin\theta)$.

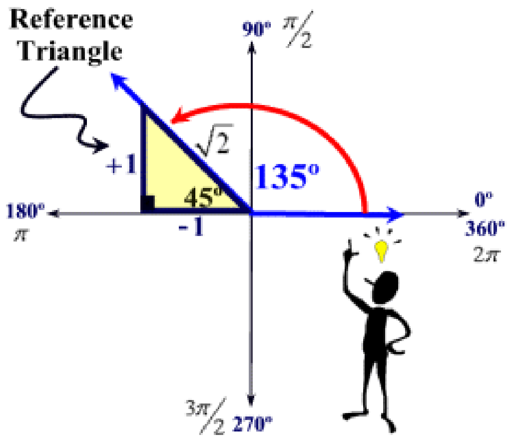


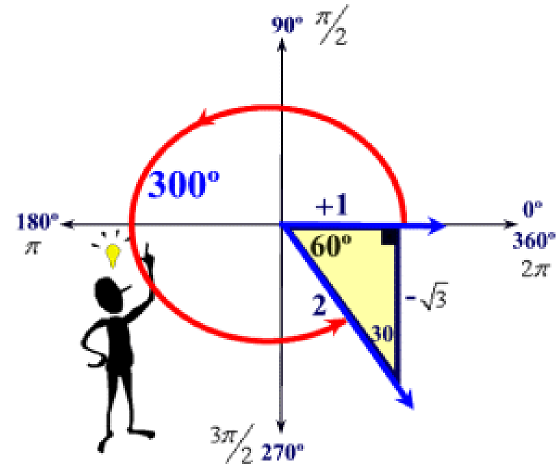
Radian Measure: One radian is the angle that intercepts an arc one unit long in a circle whose radius is 1.

π radians $= 180°$, so we can use the proportion $\dfrac{D}{R} = \dfrac{180}{\pi}$, where D is degree and R is radians, to convert.









Reciprocal Identities

$\sin(\theta) = \dfrac{1}{\csc(\theta)}$ $\cos(\theta) = \dfrac{1}{\sec(\theta)}$ $\tan(\theta) = \dfrac{1}{\cot(\theta)}$

$\csc(\theta) = \dfrac{1}{\sin(\theta)}$ $\sec(\theta) = \dfrac{1}{\cos(\theta)}$ $\cot(\theta) = \dfrac{1}{\tan(\theta)}$

Quotient Identities are also called the Tangent and Cotangent Identities

$\tan(\theta) = \dfrac{\sin(\theta)}{\cos(\theta)}$ $\cot(\theta) = \dfrac{\cos(\theta)}{\sin(\theta)}$

Sign Identities are also called **Odd and Even Function Properties**

Cosine and Secant are even, and all others are odd.

$\sin(-\theta) = -\sin(\theta)$ $\cos(-\theta) = \cos(\theta)$ $\tan(-\theta) = -\tan(\theta)$

$\csc(-\theta) = -\csc(\theta)$ $\sec(-\theta) = \sec(\theta)$ $\cot(-\theta) = -\cot(\theta)$

Cofunction Identities

$\sin\left(\dfrac{\pi}{2} - x\right) = \cos(x)$ $\tan\left(\dfrac{\pi}{2} - x\right) = \cot(x)$ $\sec\left(\dfrac{\pi}{2} - x\right) = \csc(x)$

$\cos\left(\dfrac{\pi}{2} - x\right) = \sin(x)$ $\cot\left(\dfrac{\pi}{2} - x\right) = \tan(x)$ $\csc\left(\dfrac{\pi}{2} - x\right) = \sec(x)$

$\sin(90° - x) = \cos(x)$ $\tan(90° - x) = \cot(x)$ $\sec(90° - x) = \csc(x)$

$\cos(90° - x) = \sin(x)$ $\cot(90° - x) = \tan(x)$ $\csc(90° - x) = \sec(x)$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$ $1 + \tan^2\theta = \sec^2\theta$ $1 + \cot^2\theta = \csc^2\theta$

Arc length $s = r\theta$ **Sector area** $A = \dfrac{1}{2}r^2\theta$

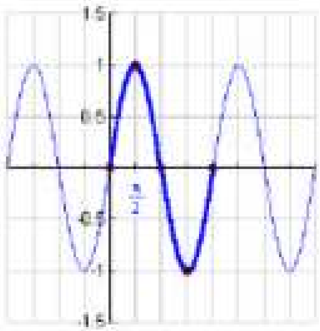
Graphs of the Six Trigonometric Functions

$y = \sin x$

Domain:
All Reals

Range:
$[-1, 1]$

Period: 2π

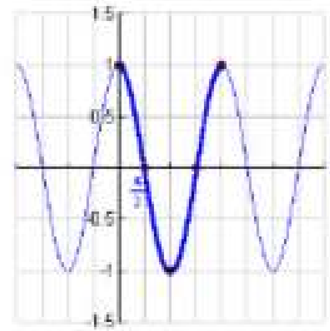


$y = \cos x$

Domain:
All Reals

Range:
$[-1, 1]$

Period: 2π

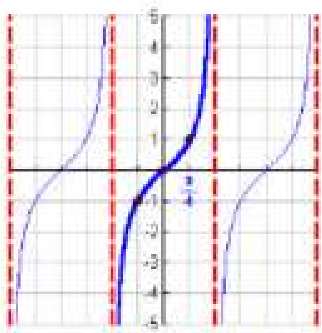


$y = \tan x$

Domain:
All $x \neq \frac{\pi}{2} + n\pi$

Range:
All Reals

Period: π

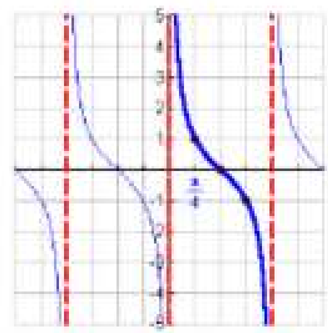


$y = \cot x$

Domain:
All $x \neq n\pi$

Range:
All Reals

Period: π

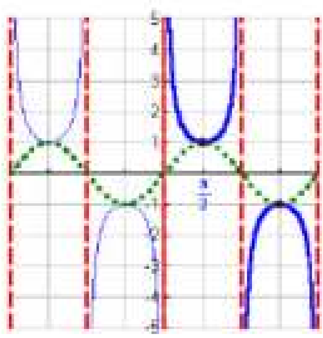


$y = \csc x$

Domain:
All $x \neq n\pi$

Range:
$(-\infty, -1] \cup [1, \infty)$

Period: 2π

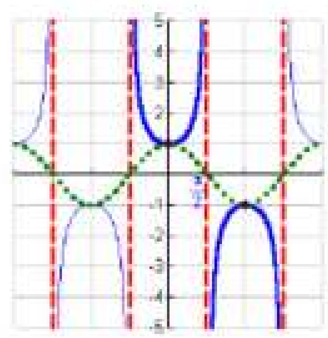


$y = \sec x$

Domain:
All $x \neq \frac{\pi}{2} + n\pi$

Range:
$(-\infty, -1] \cup [1, \infty)$

Period: 2π



Law of Sines: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$, *a, b, c* are side lengths, and *A, B, C* are angles opposite those sides.

Law of Cosines:

$a^2 = b^2 + c^2 - 2bc\cos A$ $\quad$ $b^2 = a^2 + c^2 - 2ac\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$

Note: the last version strongly resembles the Pythagorean theorem for the sides of a right triangle.

Sum and Difference Formulas

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$ $\quad$ $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

Period, Amplitude, Vertical and Horizontal Shifts, Vertical Asymptotes

	Period	Amplitude	Vertical shift	Horizontal shift	Vertical Asymptotes
$y=\sin x$	2π	1	None	None	None
$y=a\sin(bx+c)+d$	$\frac{2\pi}{b}$	a	d	$-\frac{c}{b}$	None
$y=\cos x$	2π	1	None	None	None
$y=a\cos(bx+c)+d$	$\frac{2\pi}{b}$	a	d	$-\frac{c}{b}$	None
$y=\tan x$	π	None	None	None	$\frac{\pi}{2}+k\pi, k\in Z$
$y=a\tan(bx+c)+d$	$\frac{\pi}{b}$	None	d	$-\frac{c}{b}$	$\frac{1}{b}\left(\frac{\pi}{2}-c+k\pi\right), k\in Z$
$y=\cot x$	π	None	None	None	$k\pi, k\in Z$
$y=a\cot(bx+c)+d$	$\frac{\pi}{b}$	None	d	$-\frac{c}{b}$	$\frac{1}{b}(-c+k\pi), k\in Z$
$y=\sec x$	2π	None	None	None	$\frac{\pi}{2}+k\pi, k\in Z$
$y=a\sec(bx+c)+d$	$\frac{2\pi}{b}$	None	d	$-\frac{c}{b}$	$\frac{1}{b}\left(\frac{\pi}{2}-c+k\pi\right), k\in Z$
$y=\csc x$	2π	None	None	None	$k\pi, k\in Z$
$y=a\csc(bx+c)+d$	$\frac{2\pi}{b}$	None	d	$-\frac{c}{b}$	$\frac{1}{b}(-c+k\pi), k\in Z$

Double-Angle Formulas can be derived from Sum formulas by substituting another α for β

Derive additional versions for cosine by substituting the Pythagorean Identity.

$\sin(2\alpha)=2\sin\alpha\cos\alpha$ $\qquad$ $\tan(2\alpha)=\frac{2\tan\alpha}{1-\tan^2\alpha}$

$\cos(2\alpha)=\cos^2\alpha-\sin^2\alpha$ $\qquad$ $\cos(2\alpha)=2\cos^2\alpha-1$ $\qquad$ $\cos(2\alpha)=1-2\sin^2\alpha$

Half-Angle Formulas are derived from the Power Reducing Formulas by taking square roots, then replace $2u$ by θ and u by $\frac{\theta}{2}$. You MUST know the quadrant of $\frac{\theta}{2}$ to determine + or $-$ when there's a $\pm$.

$\sin\left(\frac{\theta}{2}\right)=\pm\sqrt{\frac{1-\cos(\theta)}{2}}$ $\qquad$ $\cos\left(\frac{\theta}{2}\right)=\pm\sqrt{\frac{1+\cos(\theta)}{2}}$

$\tan\left(\frac{\theta}{2}\right)=\pm\sqrt{\frac{1-\cos(\theta)}{1+\cos(\theta)}}$ $\qquad$ $\tan\left(\frac{\theta}{2}\right)=\frac{\sin\theta}{1+\cos\theta}$ $\qquad$ $\tan\left(\frac{\theta}{2}\right)=\frac{1-\cos\theta}{\sin\theta}$

Product-to-Sum Formulas are derived by adding or subtracting two sum or difference formulas

$\sin\alpha\cos\beta=\frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$ $\qquad$ $\cos\alpha\sin\beta=\frac{1}{2}[\sin(\alpha+\beta)-\sin(\alpha-\beta)]$ continued

$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)]$ $\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$

Sum-to-Product Identities are derived by solving the linear system $\begin{cases} x = \alpha+\beta \\ y = \alpha-\beta \end{cases}$ for α and β, substituting the resulting expressions for α and β in the Product-to-Sum Formulas, and multiplying by 2.

$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$ $\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$

$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$ $\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$

Triangle Area Formulas

If a triangle has side lengths *a, b,* and *c*, with opposite angles A, B, and C, then the area of the triangle is

$Area = \frac{1}{2}bc\sin A$ $Area = \frac{1}{2}ac\sin B$ $Area = \frac{1}{2}ab\sin C$

If a triangle has side lengths *a, b,* and *c*, and $s = \frac{1}{2}(a+b+c)$, then $Area = \sqrt{s(s-a)(s-b)(s-c)}$ (Heron's)

Power Reducing Formulas are derived from the Double-Angle formulas for cosine

For example: $\cos(2u) = 2\cos^2 u - 1 = 1 - 2\sin^2 u$ Subtract 1 from both sides

$\cos(2u) - 1 = -2\sin^2 u$, then divide both sides by -2 and rearrange to get $\frac{1-\cos(2u)}{2} = \sin^2 u$

$\sin^2 u = \frac{1-\cos(2u)}{2}$ $\cos^2 u = \frac{1+\cos(2u)}{2}$ $\tan^2 u = \frac{1-\cos(2u)}{1+\cos(2u)}$

Domain and Range of Inverse Trig Functions

$y = \sin^{-1} x$ $\begin{cases} -1 \le x \le 1 \\ -\frac{\pi}{2} \le y \le \frac{\pi}{2} \end{cases}$

$y = \tan^{-1} x$ $\begin{cases} -\infty \le x \le \infty \\ -\frac{\pi}{2} \le y \le \frac{\pi}{2} \end{cases}$

$y = \csc^{-1} x$ $\begin{cases} x \le -1 \text{ or } x \ge 1 \\ -\frac{\pi}{2} \le y \le \frac{\pi}{2},\ y \ne 0 \end{cases}$

$y = \cos^{-1} x$ $\begin{cases} -1 \le x \le 1 \\ 0 \le y \le \pi \end{cases}$

$y = \cot^{-1} x$ $\begin{cases} -\infty \le x \le \infty \\ 0 \le y \le \pi \end{cases}$

$y = \sec^{-1} x$ $\begin{cases} x \le -1 \text{ or } x \ge 1 \\ 0 \le y \le \pi,\ y \ne \frac{\pi}{2} \end{cases}$

Greek alphabet

Α α *alpha*	Η η *eta*	Ν ν *nu*	Τ τ *tau*
Β β *beta*	Θ θ *theta*	Ξ ξ *xi*	Υ υ *upsilon*
Γ γ *gamma*	Ι ι *iota*	Ο o *omicron*	Φ ϕ *phi*
Δ δ *delta*	Κ κ *kappa*	Π π *pi*	Χ χ *chi*
Ε ε *epsilon*	Λ λ *lambda*	Ρ ρ *rho*	Ψ ψ *psi*
Ζ ς *zeta*	Μ μ *mu*	Σ σ *sigma*	Ω ω *omega*